SEM-2 (MATHEMATICS GENERAL)

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The solution of the equations (ie Type 1-8) may be represented in any of the following forms:

1) The cartesian form which consists of x, y and one arbitrary constant c (say).

2) The parametric form, which consists of two equations of the type $x = f_1(p, c)$, $y = f_2(p, c)$, c being an arbitrary constant and these two equations together represent the parametric equation of the general solution, in terms of the parameter p.

3) In some cases the differential equations of first order and higher degree (>1) may have solutions, which are not deducible from the complete primitive by assigning certain value to the arbitrary constant and do not contain any constant are called singular solutions.

Working Rules for finding solutions of different types of equations

a) Equations solvable for p, $p = \frac{dy}{dx}$

Let the equation $P_0 p^n + P_1 p^{n-1} + P_2 p^{n-2} + \cdots + P_n = 0$ --(1)
be solvable for p and put in the form
$(p - f_1)(p - f_2) \cdots (p - f_n) = 0$ -- (2)
where $f_1, f_2, \cdots f_n$ are some functions of x, y.

Now any relations between x and y, which makes one or more of the factors zero, is a solution of (1). Consequently each factor equated to zero produces a solution to the above equation. Let the solutions of these n-component equations be respectively, $F_1(x, y, c_1) = 0$, $F_2(x, y, c_2) = 0$,

$F_3(x, y, c_3) = 0, \cdots, F_n(x, y, c_n) = 0 \quad \text{--- (3)}$
where $c_1, c_2, \cdots, c_n$ are arbitrary constants and have any one of the infinite number of values. Thus the solutions (3) remain general, if the constants $c_1, c_2, \cdots c_n$ be replaced by an arbitrary constant c. Also the equation (1) being of first order, the general solution contains only one arbitrary constant. Thus the general solution of (1) is

$$F_1(x, y, c) \cdot F_2(x, y, c) \cdots F_n(x, y, c) = 0 \quad \text{--- (4)}$$

Ex: Solve $xy\left[\left(\frac{dy}{dx}\right)^2 - 1\right] = (x^2 - y^2)\frac{dy}{dx}$

Ans: Let $p = \frac{dy}{dx}$. Then the given equation becomes

$$xy(p^2 - 1) = (x^2 - y^2)p = 0$$
$$\Rightarrow px(py - x) + y(py - x) = 0$$
$$\Rightarrow (px + y)(py - x) = 0.$$

Either $px + y = 0$ or $x\frac{dy}{dx} + y = 0$

or $\frac{dy}{y} + \frac{dx}{x} = 0$

Integrating we get, $\log y + \log x = \log c$

$\Rightarrow xy - c = 0$, where c being an arbitrary constant.

otherwise $py - x = 0$ or $y\,dy - x\,dx = 0$

Integrating we get,

$$y^2 - x^2 = c$$

Hence the general solution of given equation is $(xy - c)(y^2 - x^2 - c) = 0$

Answer of Home work Problems (order and degree)

a) 1, 2 b) 2, 1 c) 1, 1

d) 3, 1 e) 2, 3.